The gauge invariance in the strong field approximation based treatments of light-matter interaction

A. Galstyan[1], O. Chuluunbaatar[2,3], A. Hamido[1], Yu. V. Popov[4,2], F. Mota-Furtado[5], P. O'Mahony[5] and B. Piraux[1]

[1]Université catholique de Louvain, Belgium
[2]Joint Institute for Nuclear Research, Russia
[3]National University of Mongolia, Mongolia
[4]Moscow State University, Russia
[5]University of London, United Kingdom
alexander.galstyan@uclouvain.be

Motivation

Strong field approximation (SFA) is a powerful tool widely used to describe atomic and molecular photoionisation. Usually by SFA amplitudes we mean the transition amplitudes for both length and velocity gauges (LG and VG), dependent on time and momentum. However, if one compares the results of the LG and VG calculations for real problems, one may find a severe disagreement. This problem with gauge invariance of the SFA is rather old and certainly well known. Several approaches have already been used to answer the questions:
what is the reason for this disagreement ? Should we build another theory or there is a way to make our good old SFA gauge invariant ?

Here we present a novel approach that allows one to regroup known SFA amplitudes into a set of several families. We start from the time-dependent Schrödinger equation without the Coulomb term either in LG, either in VG.

$$\left[\mathrm{i}\frac{\partial}{\partial t}+\frac{1}{2}\Delta_r-\mathrm{i}b'(t)(\vec{e}\cdot\vec{\nabla}_r)-\zeta'(t)\right]\Phi_V(\vec{r},t)=-\frac{Z}{r}\Phi_V(\vec{r},t)$$

$$\left[\mathrm{i}\frac{\partial}{\partial t}+\frac{1}{2}\Delta_r-b''(t)(\vec{e}\cdot\vec{r})\right]\Phi_L(\vec{r},t)=-\frac{Z}{r}\Phi_L(\vec{r},t)$$

We substitute one of the following ansatz, and get a first order inhomogenous differential equation. Now **we can get a full wavepacket !**

$$\boldsymbol{\Phi^1_V(\vec{r},t)=e^{-\mathrm{i}\varepsilon_0 t}\tilde{\varphi}_0(r)+F^1_V(\vec{r},t)}$$

$$\Phi^1_L(\vec{r},t)=e^{-\mathrm{i}b'(t)(\vec{e}\cdot\vec{r})-\mathrm{i}\varepsilon_0 t}\tilde{\varphi}_0(r)+F^1_L(\vec{r},t)$$

This family leads to the VGSFA

$$\Phi^2_V(\vec{r},t)=e^{\mathrm{i}b'(t)(\vec{e}\cdot\vec{r})-\mathrm{i}\varepsilon_0 t}\tilde{\varphi}_0(r)+F^2_V(\vec{r},t)$$

$$\boldsymbol{\Phi^2_L(\vec{r},t)=e^{-\mathrm{i}\varepsilon_0 t}\tilde{\varphi}_0(r)+F^2_L(\vec{r},t)}$$

This family leads to the LG SFA and PPT

SFA

- Developed by Reiss and Faisal in 1970-1980;
- S-matrix approach;
- Velocity gauge.
- Used for atoms, molecules, dielectrics.

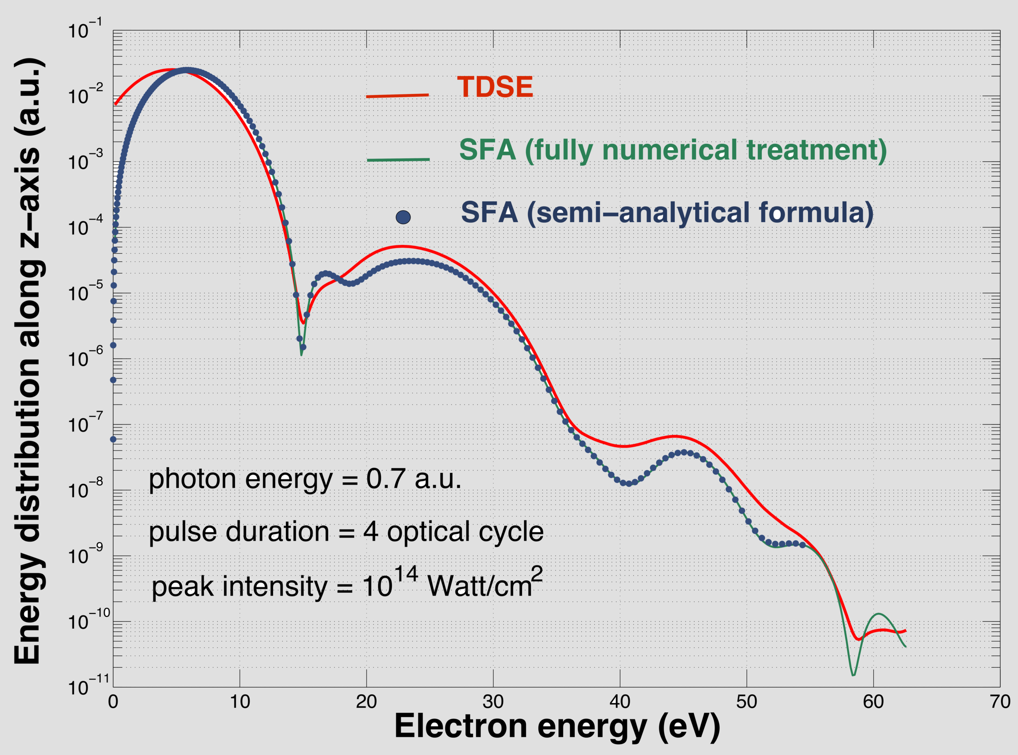
PPT

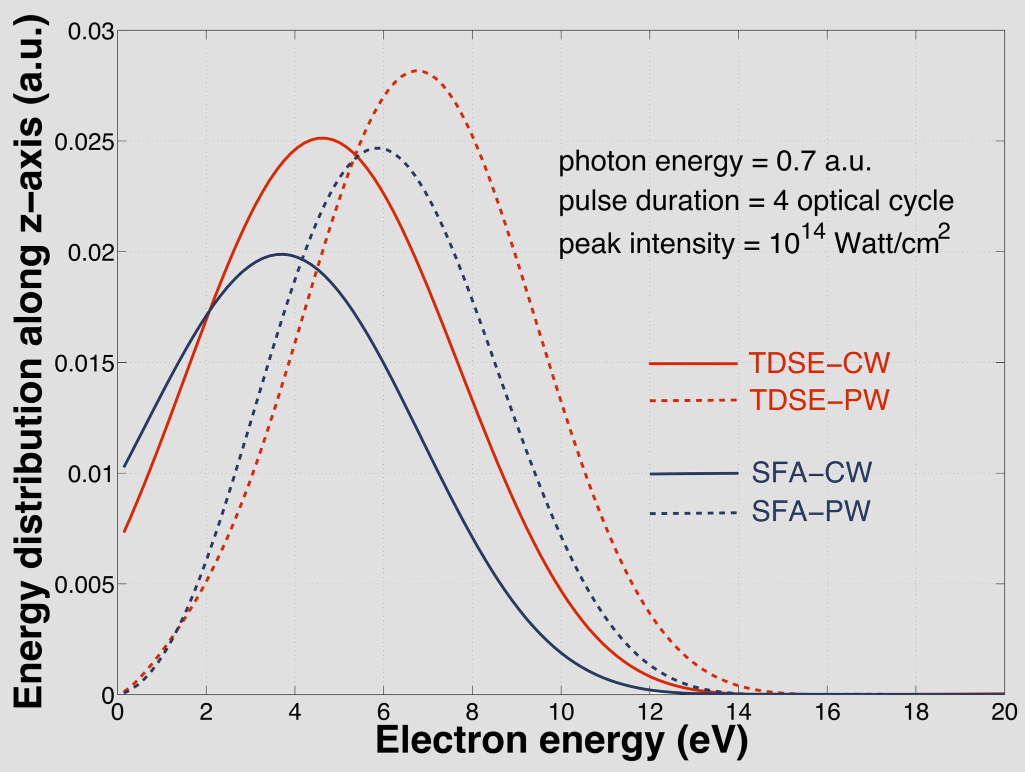
- Developed by Popov, Perelomov and Terent'ev soon after Keldysh;
- Approximate solution of Lippmann-Schwinger like equation;
- Length gauge.

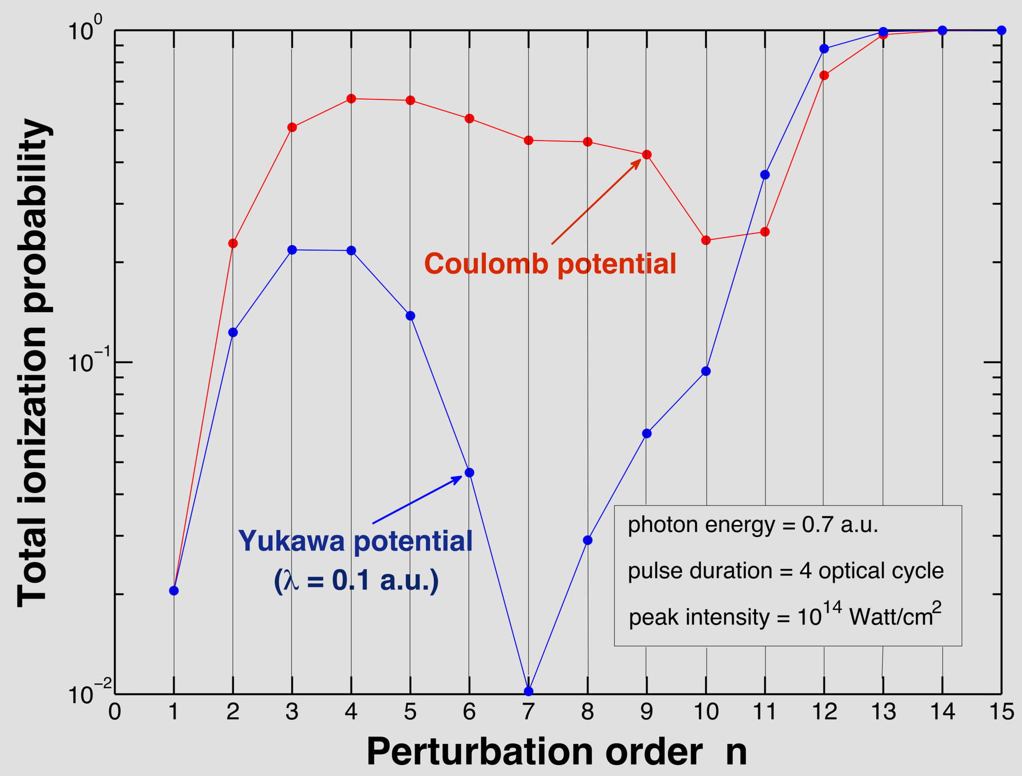
Keldysh

- Developed by Keldysh in 1964;
- Corresponds to SFA LG;
- Pioneering work on strong field ionisation model;
- Length gauge.

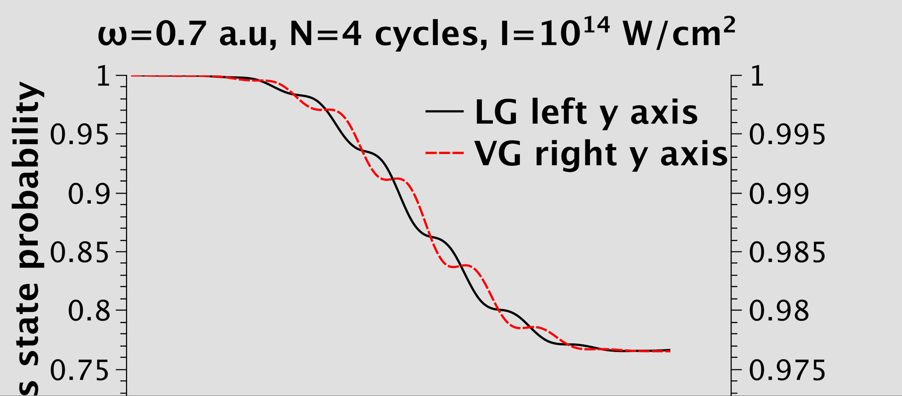
Results







$$W(t)=\frac{|\langle\varphi_0(t)|\Phi(t)\rangle|^2}{\langle\Phi(t)|\Phi(t)\rangle},\qquad P(\vec{k},t)=k\frac{|\langle\varphi_c^-(\vec{k},t)|\Phi(\vec{k},t)\rangle|^2}{\langle\Phi(t)|\Phi(t)\rangle}\frac{d^3k}{(2\pi)^3}$$



Main points