Neutrino-helium ionizing collisions: Electromagnetic contribution

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Neutrinos are very intriguing objects in particle physics. They interact very weakly and their masses are much smaller than those of the other fundamental fermions (charged leptons and quarks). In the Standard Model (SM), neutrinos are massless and have only weak interactions. However, the observation of neutrino oscillations by many experiments implies that neutrinos are massive and mixed. Therefore, the SM must be extended to account for neutrino masses. In many extensions of the SM, neutrinos also acquire electromagnetic properties through quantum loop effects (see Fig. 1). Hence, the theoretical and experimental study of neutrino electromagnetic interactions is a promising tool to search for the fundamental theory beyond the SM.

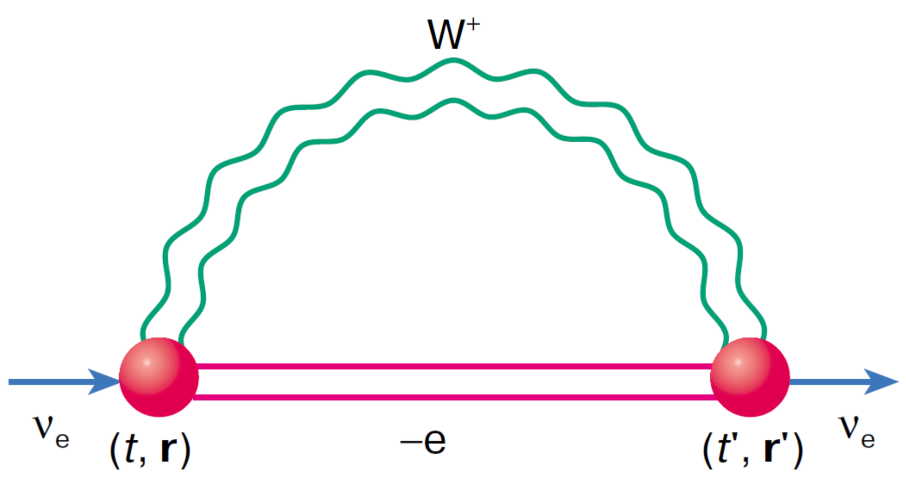

Fig. 1. *The Feynman diagram illustrating how the electron neutrino can interact with an external electromagnetic field. There is a nonzero probability that, due to the SM weak interaction, the neutrino can be "converted" into the virtual W^+ boson and electron for a short time (Δt=t′-t). These virtual charged particles interact with the electromagnetic field, thus changing the state of the neutrino.*

The most theoretically studied electromagnetic properties of neutrinos are the dipole magnetic and electric moments. The neutrino magnetic moments expected in the minimally extended SM are very small and proportional to the neutrino masses: $\mu_\nu=3\times10^{-19}\mu_B(m_\nu/1\text{ eV})$ (in units $\hbar=c=1$), with $\mu_B=e/(2m_e)$ being the electron Bohr magneton, and m_e is the electron mass. Any larger value of μ_ν can arise only from physics beyond the SM [1]. Current direct experimental searches for a magnetic moment of the electron (anti)neutrinos from reactors [2] have lowered the upper limit on its value down to $\mu_\nu<2.9\times10^{-11}\mu_B$. These ultra low background experiments use germanium crystal detectors exposed to the neutrino flux from a reactor and search for scattering events by measuring the energy deposited by the neutrino scattering in the detector. Their sensitivity to μ_ν crucially depends on lowering the threshold for the energy transfer T. This is because the electromagnetic contribution to the inclusive differential cross section for the neutrino scattering on a free electron (FE) is given by [3]

$$\frac{d\sigma_{EM}^{\rm FE}}{dT}=4\pi e^2\mu_\nu^2\left(\frac{1}{T}-\frac{1}{E_\nu}\right), \tag{1}$$

where E_ν is the incident electron energy, while that induced by weak interaction is practically constant in T (at $T<<E_\nu$):

$$\frac{d\sigma_W^{\rm FE}}{dT}=\frac{G_F^2m_e}{2\pi}\left(1+4\sin^2\theta_W+8\sin^4\theta_W\right)\left[1+O\left(\frac{T}{E_\nu}\right)\right]\approx10^{-47}\ {\rm cm^2/keV}, \tag{2}$$

where G_F is the Fermi coupling constant and θ_W is the Weinberg angle.

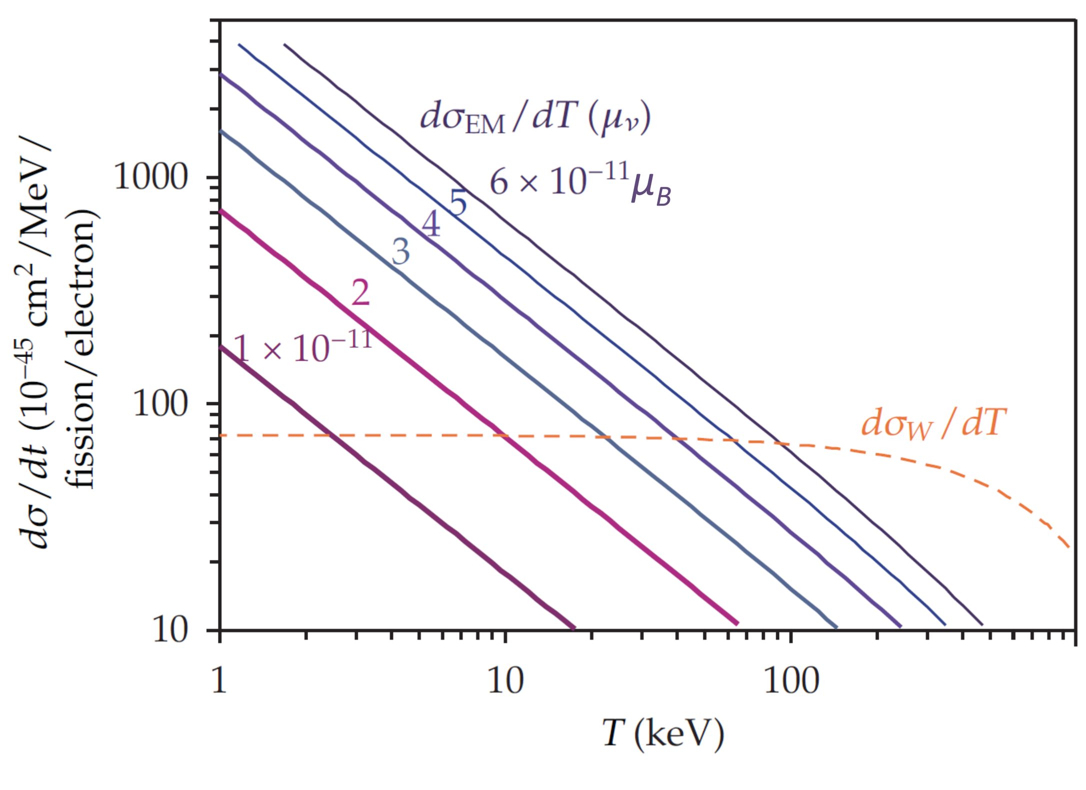

Fig. 2. *Weak (W) and electromagnetic (EM) cross sections calculated for several μ_ν values (in units of the Bohr magneton μ_B).*

The current experiments using germanium detectors have reached threshold values of T as low as few keV, where one can expect modifications of the FE formulas (1) and (2) due to the binding of electrons in the germanium atoms. Our theoretical analysis [4-6], involving the WKB and Thomas-Fermi models, has shown that the so-called stepping approximation, introduced in [7] from an interpretation of numerical data, works with a very good accuracy. According to the stepping approach, the SM and electromagnetic contributions are simply given by

$$\frac{d\sigma_W}{dT}=\frac{d\sigma_W^{\rm FE}}{dT}\sum_i n_i\theta(T-\varepsilon_i),\qquad \frac{d\sigma_{EM}}{dT}=\frac{d\sigma_{EM}^{\rm FE}}{dT}\sum_i n_i\theta(T-\varepsilon_i), \tag{3}$$

where the i sum runs over all atomic sublevels, with n_i and ε_i being their occupations and binding energies.

Recently, Martemyanov and Tsinoev [8] deduced by means of numerical calculations that the cross section $d\sigma_{EM}/dT$ for ionization of helium by neutrino impact strongly departures from the stepping approximation (3), exhibiting large enhancement relative to the FE case. They thus suggested that this finding may have an impact on searches for μ_ν, provided that its value falls within the range 10^{-13}-$10^{-12}\mu_B$. According to Martemyanov and Tsinoev, at the T values close to the ionization threshold in helium, T_I=24.5874 eV, the relative enhancement as large as almost seven orders of magnitude (see Fig. 3).

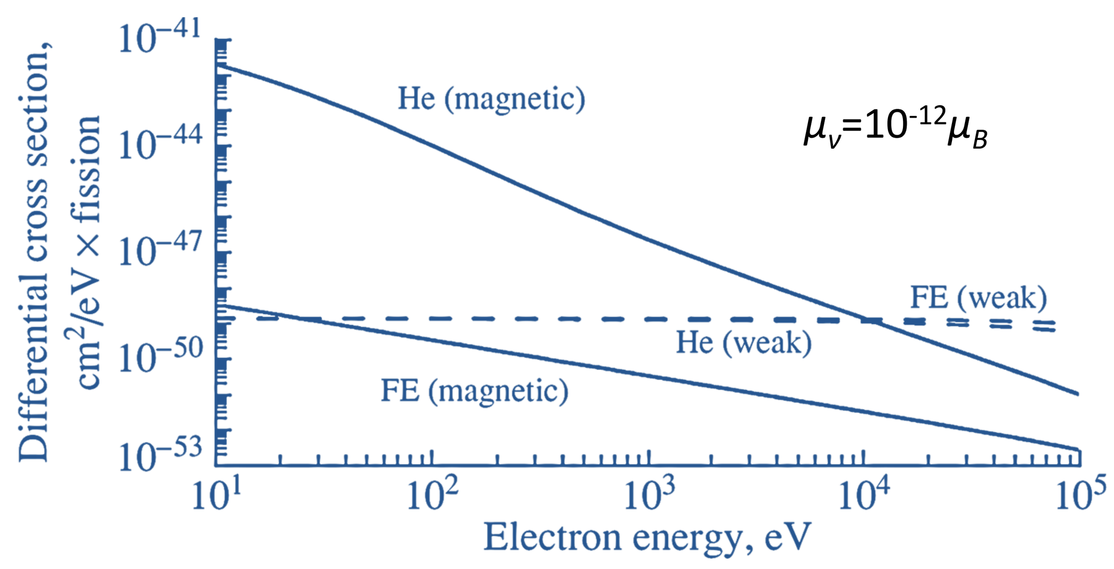

Fig. 3. *Calculations of Martemyanov and Tsinoev for $d\sigma_W/dT$ (weak) and $d\sigma_{EM}/dT$ (magnetic) in the case of ionization of helium by reactor-antineutrino impact. The figure is borrowed from Ref. [8].*

To inspect the conclusion of Martemyanov and Tsinoev, we carried out numerical estimates of the weak and electromagnetic components of the inclusive differential cross section for the ionizing neutrino-helium collision. The general formulas for these cross sections are

$$\frac{d\sigma_W}{dT}=\frac{G_F^2}{4\pi}\left(1+4\sin^2\theta_W+8\sin^4\theta_W\right)I_W(T),\qquad \frac{d\sigma_{EM}}{dT}=4\pi e^2\mu_\nu^2 I_{EM}(T), \tag{4}$$

where the functions $I_W(T)$ and $I_{EM}(T)$ are given by (when $T<<E_\nu$)

$$I_W(T)=\int_{T^2}^{4E_\nu^2}S(T,q^2)\,dq^2,\qquad I_{EM}(T)=\int_{T^2}^{4E_\nu^2}S(T,q^2)\frac{dq^2}{q^2}, \tag{5}$$

where q is the momentum transfer, and $S(T,q^2)$ is the so-called dynamical structure factor:

$$S(T,q^2)=\sum_f\left|\left\langle\Phi_f(\mathbf{r}_1,\mathbf{r}_2)\right|e^{i\mathbf{q}\cdot\mathbf{r}_1}+e^{i\mathbf{q}\cdot\mathbf{r}_2}\left|\Phi_0(\mathbf{r}_1,\mathbf{r}_2)\right\rangle\right|^2\delta(T-E_f+E_0). \tag{6}$$

Here the f sum runs over all final helium states Φ_f, with E_f being their energies. The function (6) is even in q due to the rotational symmetry of the He atom.

For evaluation of the dynamical structure factor (6) we employed simple models of the helium states that proved to be efficient in the recent theoretical analysis of the singly ionizing 100 MeV/amu C^{6+}+He collisions at small momentum transfer [9]. The ground helium state Φ_0 was approximated as

$$\Phi_0(\mathbf{r}_1,\mathbf{r}_2)=\varphi_{1s}(Z_{eff},r_1)\varphi_{1s}(Z_{eff},r_2),\qquad \varphi_{1s}(Z_{eff},r)=\sqrt{\frac{Z_{eff}^3}{\pi a_0^3}}e^{-Z_{eff}r/a_0}, \tag{7}$$

where Z_{eff}=27/16 is the effective nuclear charge, and $a_0=1/(e^2m_e)$ is the Bohr radius. The final helium state Φ_f was taken in the form

$$\Phi_f(\mathbf{r}_1,\mathbf{r}_2)=\frac{1}{\sqrt2}\left[\varphi_{\mathbf{k}}^-(Z_e,\mathbf{r}_1)\varphi_{1s}(Z=2,r_2)+\varphi_{\mathbf{k}}^-(Z_e,\mathbf{r}_2)\varphi_{1s}(Z=2,r_1)\right], \tag{8}$$

where $\varphi_{\mathbf{k}}^-$ is the Coulomb-wave state of the ejected electron with momentum **k** in a Coulomb field of charge $1\le Z_e\le2$. To avoid nonphysical effects connected with nonorthogonality of states (7) and (8), we used the Gram-Schmidt orthogonalization $|\Phi_f\rangle\to|\Phi_f\rangle-\langle\Phi_0|\Phi_f\rangle|\Phi_0\rangle$. Using Eqs. (7) and (8), we were able to perform calculations of the dynamical structure factor (6) analytically.

Following Martemyanov and Tsinoev [8], the largest effect of enhancement of the EM contribution relative to the FE case must be expected when the energy transfer approaches the ionization threshold, $T\to T_I$. Therefore, we calculated the cross sections (4) in the limiting case $T=T_I$. Since e^2=1/137, m_e=511 keV, and (for reactor antineutrinos) $E_\nu\sim1$ MeV, we have $(T_I/e^2m_e)^2\sim10^{-5}$ and $(E_\nu/e^2m_e)^2\sim10^5$. This means that the lower and upper limits of integrations in (5) can be taken as 0 and ∞, respectively, without any notable loss in accuracy. The resulting integrals were performed analytically, and the following estimates were obtained:

$$\frac{d\sigma_{EM}/dT}{d\sigma_{EM}^{\rm FE}/dT}=0.45,\qquad \frac{d\sigma_W/dT}{d\sigma_W^{\rm FE}/dT}=0.40\qquad (Z_e=1),$$
$$\frac{d\sigma_{EM}/dT}{d\sigma_{EM}^{\rm FE}/dT}=0.50,\qquad \frac{d\sigma_W/dT}{d\sigma_W^{\rm FE}/dT}=1.00\qquad (Z_e=2). \tag{9}$$

These numerical values are in qualitative agreement with the stepping approximation (3). Thus, our results disconfirm the giant enhancement of the EM contribution shown in Fig. 3.

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