New look at the strong field approximation in laser-matter interactions

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MOTIVATIONS:

-- People say, that SFA calculations are gauge dependent.

Is it correct ?

-- Papers are published with the statements, that SFA in the length gauge is in better agreement with the experiment, than that in the velocity gauge, or vice versa.

What is in a reality?

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Definitions for laser pulse:

$$\frac{1}{c}\vec{A}(t) = -b'(t)\vec{e}, \quad \vec{E}(t) = b''(t)\vec{e}., \quad 0 \le t \le T$$

$$b'(t) = \frac{1}{\omega_0}\sqrt{\frac{I}{I_0}}\sin^2(\pi\frac{t}{T})\sin(\omega_0 t), \; T = \frac{2\pi N}{\omega_0}$$

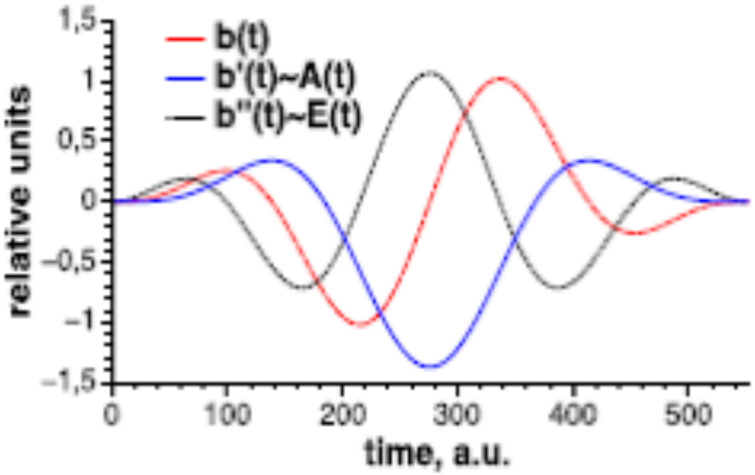

Fig. 1: The curves $b(t)$, $b'(t)$ and $b''(t)$ in a relative scale. $\omega_0 = 0.0228$ a.u. and $I = 10^{14}$ W/cm^2.

Time dependent Schroedinger equation

$$\left[i\frac{\partial}{\partial t}+\frac{1}{2}\triangle_r+\frac{Z}{r}-ib'(t)(\vec{e}\cdot\vec{\nabla}_r)-\zeta'(t)\right]\times$$

$$\tilde{\Phi}_V(\vec{r},0) = \tilde{\varphi}_0(r)$$

$$\tilde{\Phi}_V(\vec{r},t)=0,\quad \zeta(t)=(1/2)\int_0^t d\xi[b'(\xi)]^2 \quad (1)$$

V - gauge

$$\tilde{\Phi}_L(\vec{r},t)=e^{-ib'(t)(\vec{e}\cdot\vec{r})}\tilde{\Phi}_V(\vec{r},t), \qquad (2)$$

LV - transformation

$$\left[i\frac{\partial}{\partial t}+\frac{1}{2}\triangle_r+\frac{Z}{r}-b''(t)(\vec{e}\cdot\vec{r})\right]\tilde{\Phi}_L(\vec{r},t)=0. \qquad (3)$$

L - gauge

Lipmann-Schwinger (Dyson) equation

$$\tilde{\Phi}_V(\vec{r},t) = i\int d^3r'\,\tilde{G}_V(\vec{r},t;\vec{r}\,',0)\tilde{\varphi}_0(\vec{r}\,') -$$

$$Z\int_0^t dt' \int \frac{d^3r'}{r'}\tilde{G}_V(\vec{r},t;\vec{r}\,',t')\tilde{\Phi}_V(\vec{r}\,',t') = I_0^V + I^V. \qquad (4)$$

$$\tilde{G}_V(\vec{r},t;\vec{r}\,',t') = -i\theta(t-t')$$

$$\int \frac{d^3p}{(2\pi)^3}\tilde{\chi}_V(\vec{r},\vec{p},t)\tilde{\chi}_V^*(\vec{r}\,',\vec{p},t'), \qquad (5)$$

Green's function

$$\tilde{\chi}_V(\vec{r},\vec{p},t) = e^{[i\vec{p}\cdot\vec{r} - i(p^2/2)t + ib(t)(\vec{e}\cdot\vec{p}) - i\zeta(t)]}$$

V-Volkov wave

Lipmann-Schwinger (Dyson) equation

- V- and L-Volkov waves are connected by the same phase factor like in (2);
- The integral Lippmann-Schwinger equation for (2) writes by replacing in (4) and (5) V→L;
- The first term I_0^V in (4) could be chosen like the starting (zeroth) term for the Born series following from (4).

Born terms for L- and V-equation are related as

$$I_n^L = e^{-ib'(t)(\vec{e}\cdot\vec{r})} I_n^V \qquad (6)$$

Eq. (4) is correct for an arbitrary initial wave packet, NOT for the hydrogen ground state!

V-gauge ansatz

$$\tilde{\Phi}_V(\vec{r}, t) = e^{-i\varepsilon_0 t}\tilde{\varphi}_0(r) + \tilde{F}_V(\vec{r}, t). \qquad (7)$$

$$\tilde{F}_V(\vec{r}, t) = \tilde{F}_{V0}(\vec{r}, t) + iZ \int \frac{d^3p}{(2\pi)^3}\tilde{\chi}_V(\vec{r}, \vec{p}, t)\times$$

$$\int_0^t dt' \int \frac{d^3r'}{r'}\tilde{\chi}_V^*(\vec{r}\,', \vec{p}, t')\tilde{F}_V(\vec{r}\,', t') \qquad (8)$$

LS-equation

$$\tilde{F}_{V0}(\vec{r}, t) = \int \frac{d^3p}{(2\pi)^3}\tilde{\chi}_V(\vec{r}, \vec{p}, t) \int_0^t dt' \int d^3r' \times$$

$$\tilde{\chi}_V^*(\vec{r}\,', \vec{p}, t')\left[b'(t')e^{-i\varepsilon_0 t'}(\vec{e}\cdot\vec{\nabla}_{r'})\tilde{\varphi}_0(r') - i\zeta'(t')\right]. \qquad (9)$$

V-SFA (first Born, Bauer)

V-gauge ansatz

The corresponding L-gauge ansatz

$$\Phi_L(\vec{r},t) = e^{-ib'(t)(\vec{e}\cdot\vec{r})-i\varepsilon_0 t}\bar{\varphi}_0(r) + F_L'(\vec{r},t) \qquad (10)$$

gives the same V-SFA.

Formula (10) generates the gauge-invariant family.

L-gauge ansatz

$$\bar{\Phi}_L(\vec{r},t) = e^{-i\varepsilon_0 t}\bar{\varphi}_0(r) + \bar{F}_L(\vec{r},t). \qquad (11)$$

$$\bar{F}_L(\vec{r},t) = \bar{F}_{L0}(\vec{r},t) + iZ\int\frac{d^3p}{(2\pi)^3}\bar{\chi}_L(\vec{r},\vec{p},t)\times$$

$$\int_0^t dt' \int \frac{d^3r'}{r'}\bar{\chi}_L^*(\vec{r}\,',\vec{p},t')\bar{F}_L(\vec{r}\,',t') \qquad (12)$$

LS-equation

$$\bar{F}_{L0}(\vec{r},t) = -i\int\frac{d^3p}{(2\pi)^3}\bar{\chi}_L(\vec{r},\vec{p},t)\int_0^t dt'\int d^3r'\times$$

$$\bar{\chi}_L^*(\vec{r}\,',\vec{p},t')b''(t')e^{-i\varepsilon_0 t'}(\vec{e}\cdot\vec{r}\,')\bar{\varphi}_0(r') \qquad (13)$$

L-SFA (first Born, Keldysh)

L-gauge ansatz

The ansatz (11) generates now **another gauge-invariant family**. They can be compared only with the exact solutions of the corresponding TDSE and the phase connection of the L and V wave packets.

X = a+b = c+d

a+b – one family, c+d – another family

Calculation of energy spectra

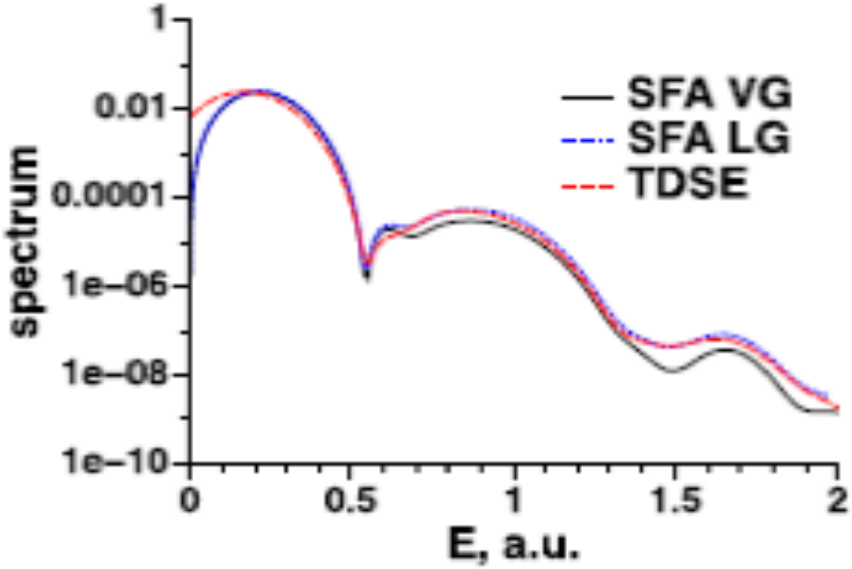

Fig. 2: Energy spectrum of ejected electrons, forward scattering ($\theta_e = 0$). SFA curves are normalized to TDSE. The field parameters: $\omega_0 = 0.7$ a.u., $\sqrt{I/I_0} = 0.05$, $N = 4$.

Calculation of energy spectra

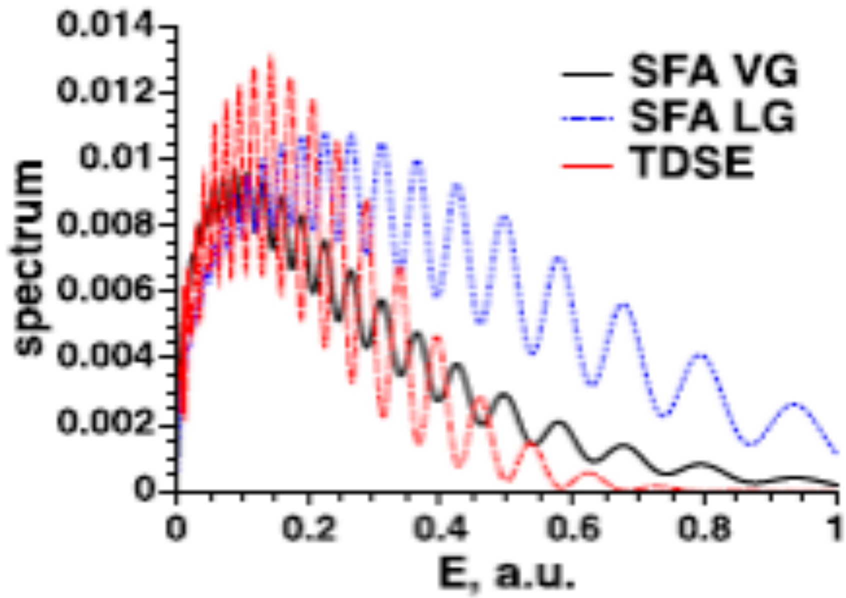

Fig. 3: The same like in Fig. 2, but $\omega_0 = 0.0228$ a.u. and $N = 2$.

Conclusions

SFA is the first Born term, if the Coulomb potential is considered like a perturbation. Correct perturbation means that higher terms are smaller than the first one. This is a domain of high frequencies and low intensities.

Conclusions

Correct gauge transformation of the chosen ansatz gives the gauge invariant Born series, which can be called as the gauge invariant family. The V-SFA and L-SFA are related to DIFFERENT families. They are different and CANNOT be gauge invariant by definition. But the full Born series of different families are gauge invariant.

КОНЕЦ

Спасибо за внимание !